Deciphering the Archaeological Record:
Cosmological Imprints of Non-Minimal Dark Sectors

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University of Wisconsin

Wednesday, November 13th, 2019
dark matter = ??
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• sits at the border between particle physics/astrophysics/cosmology
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*unfortunately, not much known:*

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- **production** mechanism? (thermal/non-thermal?)
- one species? or many components?
- interactions with SM? within dark sector itself?
- what **dynamics** is involved in establishing DM today?

[source: Science/AAS]
In this talk:
We are interested in how dark matter drives cosmological structure.
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We are interested in how dark matter drives **cosmological structure**.

![Diagram](image-url)

deep inside the early universe, the dark matter phase-space distribution $f(p)$ determines the matter power spectrum $P(k)$. 
In this talk:
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To what extent can we find signatures or patterns in $P(k)$ tell us about early universe dynamics that produced the dark matter?
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To what extent can we find signatures or patterns in $P(k)$ tell us about early universe dynamics that produced the dark matter?
PART I

early-universe dynamics $\rightarrow$ DM phase-space distribution
In general, once the dark matter is produced in the early universe its properties are described by its phase-space distribution $f(\vec{x}, \vec{p}, t) \approx f(\vec{p}, t)$: homogeneity/isotropy.

- Number density:
  $$n(t) = g_{\text{int}} \int \frac{d^3 p}{(2\pi)^3} f(p, t)$$

- Energy density:
  $$\rho(t) = g_{\text{int}} \int \frac{d^3 p}{(2\pi)^3} E f(p, t)$$

- Pressure:
  $$P(t) = g_{\text{int}} \int \frac{d^3 p}{(2\pi)^3} \frac{p^2}{3E} f(p, t)$$

The equation of state:
$$w(t) = \frac{P(t)}{\rho(t)}$$

⇒ The distribution $f(p, t)$ is the central quantity in understanding cosmological properties of the dark sector.
Early Dynamics → DM Momentum Distributions

- It is important to understand how \( f(p) \) evolves in an FRW background:

\[
p(t) = p(t') \frac{a(t')}{a(t)}
\]

Redshifting gives

\[
\frac{d \log p}{dt} = -H(t)
\]

Hubble parameter
Early Dynamics \rightarrow \text{DM Momentum Distributions}

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\[ \text{redshifting} \]

$\Rightarrow$ time-evolution corresponds to overall shifts in $\log p$

$$N(t) \equiv a^3 n \propto a^3 \int d^3 p f(p, t) = 4\pi \int d \log p (pa)^3 f(p)$$

\[ \text{comoving number density} \]
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\[ \text{comoving number density} \]

motivates a definition

\[
g(p, t) \equiv a(t)^3 p^3 f(p, t)
\]

such that \( N \propto \int d \log p g(p) \).
Early Dynamics \(\rightarrow\) DM Momentum Distributions

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Under time-evolution

\[g(p(t), t) = g(p(t'), t'), \text{ i.e., the shape is fixed, but it shifts in } \log p, \text{ as if carried along by a cosmological \textquoteleft\textquoteleft conveyor belt\textquoteright\textquoteleft\textquoteright\textquoteright.}\]

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- Hubble parameter

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Early Dynamics $\longrightarrow$ DM Momentum Distributions

- Allowing interactions, non-thermal production could potentially yield interesting scenarios:

\[
\log p
\]

flow of conveyor belt
Early Dynamics → DM Momentum Distributions

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![Diagram showing momentum distributions with deposits at different times](image)
Early Dynamics → DM Momentum Distributions

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\[
\log p
\]

deposit at \(t_1\)

deposit at \(t_2\)

flow of conveyor belt
Early Dynamics \[\rightarrow\] DM Momentum Distributions

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\[\Rightarrow\] after deposits completed, resulting distribution can be highly non-trivial and even *multi-modal.*
Early Dynamics → DM Momentum Distributions

- Allowing interactions, non-thermal production could potentially yield interesting scenarios:

\[
g(p) = \int dt' \Delta \left( p \frac{a(t)}{a(t')}, t' \right)
\]

accumulation of deposits with profile \( \Delta(p, t) \):

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g(p) = \int dt' \Delta \left( p \frac{a(t)}{a(t')}, t' \right)
\]

⇒ after deposits completed, resulting distribution can be highly non-trivial and even **multi-modal**.
what properties naturally give rise to such deposits?

If the dark sector contains an ensemble of states with different masses, then these deposits arise naturally from **intra-ensemble decays** (decays within dark sector)
Early Dynamics $\rightarrow$ DM Momentum Distributions

- To consider how this works, take a three-state system with $m_2 > m_1 > m_0$, and only the heaviest initially produced (for simplicity).
Early Dynamics \(\rightarrow\) DM Momentum Distributions

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![Diagram showing g(p) and redshift](image-url)
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BASIC OBSERVATIONS:

$2 \rightarrow 1 + 0$: daughter packets get extra kinetic energy and width ($\Delta p$) compared to parent packet.
Early Dynamics $\rightarrow$ DM Momentum Distributions

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resulting distribution $g(p)$ is superposition of deposits from two separate decay chains—carries imprints of the early decay dynamics.
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resulting distribution $g(p)$ is superposition of deposits from two separate decay chains—carries imprints of the early decay dynamics

but what precisely sets the detailed shape of each packet?
Let us investigate the process of a single decay in detail:

\[ g_P(p) \] for parent and \[ g_D(p) \] for daughters.
Early Dynamics \(\rightarrow\) DM Momentum Distributions

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Early Dynamics $\rightarrow$ DM Momentum Distributions

- Let us investigate the process of a single decay in detail:

\[ P \rightarrow D \]

\[ g_P(p) \rightarrow g_D(p) \]

\[ \text{PARENT} \rightarrow \text{DAUGHTERS} \]

\[ \log p \rightarrow \log p \]

\[ \text{redshift} \rightarrow \text{decay at } t_A \rightarrow \text{decay at } t_B \]
Early Dynamics $\rightarrow$ DM Momentum Distributions

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Early Dynamics → DM Momentum Distributions

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$$g_P(p)$$

PARENT

$$g_D(p)$$

DAUGHTERS
Early Dynamics $\rightarrow$ DM Momentum Distributions

- This detailed analysis allows us to **infer** properties of the **parent packet**, simply by examining the **properties of the daughter packet**.
Early Dynamics $\rightarrow$ DM Momentum Distributions

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**EXAMPLE:**
In our analysis we have found that

- **leftward tilt** (positive skew): relativistic at production
- **rightward tilt** (negative skew): non-relativistic at production
Early Dynamics \(\rightarrow\) DM Momentum Distributions

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**EXAMPLE:**
In our analysis we have found that

- **Leftward tilt** (positive skew) \(\rightarrow\) relativistic at production
- **Rightward tilt** (negative skew) \(\rightarrow\) non-relativistic at production

We could have a narrow daughter packet \((i.e., \Delta p \ll m \text{ and } \Delta p \ll \langle p \rangle)\) with a parent packet that is either
  - relativistic with a close-to-marginal decay
  - non-relativistic with a far-from-marginal decay

but the tilt/skewness allows us to distinguish.
Early Dynamics $\rightarrow$ DM Momentum Distributions

- We can go even further and map out all of the correlations:

<table>
<thead>
<tr>
<th>Daughter packet</th>
<th>Parent packet</th>
<th>Decay near marginality?</th>
<th>Decay near “relative marginality”?</th>
</tr>
</thead>
<tbody>
<tr>
<td>rel? (max $p$)</td>
<td>rel at production?</td>
<td>rel</td>
<td>rel~</td>
</tr>
<tr>
<td>tilt</td>
<td>rel at decay?</td>
<td>rel$\gg$</td>
<td>rel$\gg$</td>
</tr>
<tr>
<td>width $\Delta p/m$</td>
<td>$\mathcal{O}(1)$</td>
<td>rel</td>
<td>non-rel</td>
</tr>
<tr>
<td>relative width $\Delta p/\langle p \rangle$</td>
<td>narrow</td>
<td>rel$\gg$</td>
<td>rel$\gg$</td>
</tr>
<tr>
<td></td>
<td>leftward</td>
<td>wide</td>
<td>$\mathcal{O}(1)$</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>$\mathcal{O}(1)$</td>
</tr>
<tr>
<td>$p \gg m$</td>
<td>rightward</td>
<td>wide</td>
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<tr>
<td>$p \sim m$</td>
<td>leftward</td>
<td>narrow</td>
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</tr>
</tbody>
</table>

and even apply these to the **constituent parts** of multi-modal distributions.
To verify that these features appear we need to (numerically) solve the Boltzmann system:

\[
\frac{\partial f_\ell(p_\ell, t)}{\partial t} = H(t)p_\ell \frac{\partial f_\ell}{\partial p_\ell} + \frac{C[f]}{\sqrt{p_\ell^2 + m_\ell^2}}
\]

for the three-state system.
Early Dynamics → DM Momentum Distributions

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\]

• Assume that \( f_\ell \ll 1 \) and we have an initially populated thermal distribution \( g_2(p) \)

• Everything else is determined by the decay widths \( \Gamma_{ij}^\ell \) and the Hubble parameter \( H \)
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Everything else is determined by the decay widths $\Gamma_{ij}^\ell$ and the Hubble parameter $H$. 

\begin{align*}
m_2 &= 7m_0 \\
m_1 &= 3m_0 \\
\{0, 10, 0.1\} \\
\{0, 10, 10\} \\
\{0, 10, 100\} \\
\{10, 10, 0.1\} \\
\{\Gamma_{00}^2, \Gamma_{11}^2, \Gamma_{00}^1\}/H \\
p/m_0 \\
0 &\quad 0.1 &\quad 1
\end{align*}
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Early Dynamics $\rightarrow$ DM Momentum Distributions

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I Early Dynamics → DM Momentum Distributions

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redshifting

collision terms

for the three-state system.

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Early Dynamics \(\longrightarrow\) DM Momentum Distributions

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\]

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In the only case with competing decay chains we find multi-modal distributions are produced.
PART II
DM phase-space distribution $\rightarrow$ matter power spectrum
INITIAL CONDITIONS
(primordial perturbations)
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(primordial perturbations)

POWER SPECTRA

\[ T^2(k) \equiv \frac{P(k)}{P_{CDM}(k)} \]
INITIAL CONDITIONS (primordial perturbations)

$g(p)$

evolve perturbations (using CLASS software)

POWER SPECTRA

$T^2(k) \equiv \frac{P(k)}{P_{CDM}(k)}$
Momentum Distributions → Matter Power Spectra
(Cold) dark matter drives the growth of structure
(Cold) dark matter drives the growth of structure

\[ \ddot{\delta} + 2H \dot{\delta} + \frac{k^2}{a^2} c_s^2 \delta - \frac{4\pi G \rho \delta}{a^2} = 0 \]

DM density perturbations

\[ \delta \equiv \frac{\delta \rho_{DM}}{\rho_{DM}} \]
• (Cold) **dark matter** drives the growth of **structure**

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\ddot{\delta} + 2H \dot{\delta} + \frac{k^2}{a^2} c_s^2 \delta - 4\pi G \rho \delta = 0
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\[
\delta \equiv \frac{\delta \rho_{DM}}{\rho_{DM}}
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and with \(c_s^2 \neq 0\) small perturbations \(\frac{k}{a} > \sqrt{\frac{3}{2}} \frac{H}{c_s}\) do not grow
(Cold) dark matter drives the growth of structure

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A standard approach is to define a free-streaming horizon

\[ k_{FSH}^{-1} \equiv \int_{t_{prod}}^{t_{now}} dt \frac{\langle v(t) \rangle}{a(t)} \]

as a benchmark for the scale below which structure is suppressed.
(Cold) **dark matter** drives the growth of **structure**

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relies on **averaging** over DM distribution

will **fail** for multi-modal \(g(p)\)

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as a benchmark for the scale below which structure is suppressed.

We’ll consider a different approach...
Momentum Distributions $\rightarrow$ Matter Power Spectra

Our Approach:
Our Approach:

- We begin by considering *momentum slices* through the distribution:

\[
k_{FSH}(p) \equiv \left[ \int_{t_{prod}}^{t_{now}} \frac{p/a(t)}{\sqrt{p^2/a(t)^2 + m^2 a(t)}} \frac{dt}{a(t)} \right]^{-1}
\]
II Momentum Distributions \(\rightarrow\) Matter Power Spectra

Our Approach:

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• We take this \(k_{FSH}(p)\) relation to define a *mapping* between \(p\) [of the dark-matter distribution \(g(p)\)] and \(k\) [of the power spectrum \(P(k)\)].
Our Approach:

• We begin by considering *momentum slices* through the distribution:

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k_{\text{FSH}}(p) \equiv \left[ \int_{t_{\text{prod}}}^{t_{\text{now}}} \frac{p/a(t)}{\sqrt{p^2/a(t)^2 + m^2}} \frac{dt}{a(t)} \right]^{-1}
\]

• We take this \( k_{\text{FSH}}(p) \) relation to define a *mapping* between \( p \) [of the dark-matter distribution \( g(p) \)] and \( k \) [of the power spectrum \( P(k) \)].

• In other words, we identify \( k_{\text{FSH}}(p) \) with \( k \) and consider \( g(p) \) as having a corresponding profile in \( k \)-space:

\[
\tilde{g}(k) \equiv g(k_{\text{FSH}}^{-1}(k)) |\mathcal{J}(k)|
\]
Our Approach:

- We begin by considering \textit{momentum slices} through the distribution:

\[
k_{\text{FSH}}(p) \equiv \left[ \int_{t_{\text{prod}}}^{t_{\text{now}}} \frac{p/a(t)}{\sqrt{p^2/a(t)^2 + m^2 a(t)}} \frac{dt}{a(t)} \right]^{-1}
\]

- We take this \(k_{\text{FSH}}(p)\) relation to define a \textit{mapping} between \(p\) [of the dark-matter distribution \(g(p)\)] and \(k\) [of the power spectrum \(P(k)\)].

- In other words, we identify \(k_{\text{FSH}}(p)\) with \(k\) and consider \(g(p)\) as having a corresponding profile in \(k\)-space:

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\tilde{g}(k) \equiv g\left(k_{\text{FSH}}^{-1}(k)\right) |\mathcal{J}(k)|
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\textit{inverse function of free-streaming horizon}
II. Momentum Distributions $\rightarrow$ Matter Power Spectra

Our Approach:

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$$\tilde{g}(k) \equiv g\left(k_{FSH}^{-1}(k)\right) |J(k)|$$

which retains $\mathcal{N} = \int d\log p \ g(p) = \int d\log k \ \tilde{g}(k)$. 

• Inverse function of free-streaming horizon

• Jacobian
Momentum Distributions → Matter Power Spectra
we are finally equipped to ask:

Can we conjecture the relationship

$$\tilde{g}(k) \leftrightarrow T^2(k)$$

between distributions/power spectra?
we are finally equipped to ask:

Can we conjecture the *relationship*

\[ \tilde{g}(k) \leftrightarrow T^2(k) \]

between distributions/power spectra?

let’s do a bit of exploring...
• For simplicity, consider a simple uni-modal dark-matter phase space distribution $g(p)$.

• We vary the fraction of dark matter abundance $r \equiv \Omega / \Omega_{DM}$ carried by $g(p)$ and assume that the rest is pure CDM.
II Momentum Distributions → Matter Power Spectra

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**BASIC OBSERVATIONS:**

- no power suppression until we approach where $\tilde{g}(k)$ is concentrated
Momentum Distributions ➔ Matter Power Spectra

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- no power suppression until we approach where \( \tilde{g}(k) \) is concentrated

- more \( \tilde{g}(k) \) abundance (larger \( r \))
  \( \Rightarrow \) more suppression/steeper slope

\[ T^2(k) = \frac{g(k, t_{\text{now}})}{N} \]

\[ g(p, t_{\text{now}}) / N \]

\[ g(p) \]

\( \sigma = 0.4 \)

\( r = 1.0 \)

\( r = 0.9 \)

\( r = 0.75 \)

\( r = 0.6 \)

\( r = 0.45 \)

\( r = 0.3 \)

\( r = 0.15 \)
For simplicity, consider a simple uni-modal dark-matter phase space distribution $g(p)$.

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- more $\tilde{g}(k)$ abundance (larger $r$)

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- acoustic oscillations begin to show as $\tilde{g}(k)$ carries close to full DM abundance

\[ T^2(k) \equiv \frac{P(k)}{P_{\text{CDM}}(k)} \]
II Momentum Distributions \(\rightarrow\) Matter Power Spectra

- Now fix the \(g(p)\) abundance (and its \(\langle p \rangle_{\text{now}}\)) but vary the width of the distribution.
Now fix the $g(p)$ abundance (and its $\langle p \rangle_{\text{now}}$) but vary the width of the distribution.

\[
\Omega = \frac{3}{4} \Omega_{\text{DM}} \\
\langle p \rangle_{\text{now}} = 10^{-6} m
\]

\[
\tilde{g}(k, t_{\text{now}})/N
\]

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T^2(k) = \frac{P(k)}{P_{\text{CDM}}(k)}
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• as we **widen** the distribution:
  ○ slope of $T^2(k)$ changes **more slowly**
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  - HOWEVER, the slope of $T^2(k)$ itself remains the same at large $k$
II Momentum Distributions → Matter Power Spectra

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- suggests **relationship** between “accumulated abundance” in $\tilde{g}(k)$ and slope of $T^2(k)$
  [{i.e., sweeping to larger $k$, more accumulated abundance ⇒ slope increasingly steep}]
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$\tilde{g}(k)$ abundance correlates **not** with suppression of $T^2(k)$ but with its **slope**.
• Do these observations survive for a more complicated $g(p)$ distribution?

• Let’s examine two peaks and vary their relative abundances.
Momentum Distributions \( \rightarrow \) Matter Power Spectra

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II Momentum Distributions → Matter Power Spectra

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Momentum Distributions → Matter Power Spectra

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Next, let’s quantify these observations....
PART III
The “Archaeological” Inverse Problem
III The “Archaeological” Inverse Problem

- At any particular $k$: the accumulated abundance is

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k') d\log k'}{\int_{-\infty}^{+\infty} \tilde{g}(k') d\log k'},$$

or equivalently the fraction of our DM which is effectively “hot” (i.e., free-streaming).
The “Archaeological” Inverse Problem

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“hot fraction” function

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- Our claim is that the slope of $T^2(k)$ is directly related to $F(k)$

\[ F(k) \approx \eta \left( \left| \frac{d\log T^2}{d\log k} \right| \right) \]

some as-yet unknown function
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so that (differentiating) we find

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phase-space distribution
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transfer function slope
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phase-space distribution  transfer function slope transfer function curvature
III The “Archaeological” Inverse Problem

- Using our earlier results we can implicitly determine the function $\eta$:

$$\left| \frac{d \log T^2}{d \log k} \right| \approx F^2(k) + \frac{3}{2} F(k)$$
III The “Archaeological” Inverse Problem

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and therefore we can finally state our conjectured relation:

$$\tilde{g}(k) \approx \frac{1}{2} \left( \frac{9}{16} + \left| \frac{d \log T^2}{d \log k} \right| \right)^{-1/2} \left| \frac{d^2 \log T^2}{(d \log k)^2} \right|$$

With this relation we can “resurrect” the DM distribution $\tilde{g}(k)$ from the transfer function $T^2(k)$.
The “Archaeological” Inverse Problem

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A technical aside:

Our conjecture has a built-in assumption that $d^2 \log T^2(k)/(d \log k)^2$ is negative-semidefinite. This tends to cover cases in which $\tilde{g}(k)$ is relatively “clustered,” regardless of the complexity of its shape.
An Illustrative Model of Multi-Component Decay Chains
Consider a model with $N + 1$ real scalars $\{\phi_0, \phi_1, \ldots \phi_N\}$ with a mass spectrum

$$m_\ell = m_0 + \ell^\delta \Delta m$$

and Lagrangian

$$\mathcal{L} = \sum_{\ell=0}^{N} \left( \frac{1}{2} \partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell} \sum_{j=0}^{i} c_{\ell i j} \phi_\ell \phi_i \phi_j \right) + \cdots$$
Illustrative Model of Multi-Component Decay Chains

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• Let’s parameterize the trilinear couplings in a useful way for our study:

$$c_{\ell ij} = \mu R_{\ell ij} \left( \frac{m_\ell - m_i - m_j}{\Delta m} \right)^r \left( 1 + \frac{|m_i - m_j|}{\Delta m} \right)^{-s} \Theta(m_\ell - m_i - m_j)$$
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where $\mu$ is the overall mass scale, $R_{\ell ij}$ is the counting factor gap between parents and daughters, and $\Theta$ is the step function.
Illustrative Model of Multi-Component Decay Chains

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energy released

- $r > 0$: maximally exothermic decays
- $r < 0$: minimally exothermic decays
Illustrative Model of Multi-Component Decay Chains

Consider a model with \( N + 1 \) real scalars \( \{\phi_0, \phi_1, \ldots, \phi_N\} \) with a mass spectrum

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- \( r > 0 \): maximally exothermic decays
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- \( s > 0 \): maximally symmetric daughters
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Illustrative Model of Multi-Component Decay Chains

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\]

**Fix parameters:**
- \( N = 9 \)
- \( \delta = 1 \)
- \( \Delta m = 2m_0 \)
- \( \mu = 0.1m_0 \)

**Energy released**
- \( r > 0 \): maximally exothermic decays
- \( r < 0 \): minimally exothermic decays

**Symmetry in decay**
- \( s > 0 \): maximally symmetric daughters
- \( s < 0 \): minimally symmetric daughters
Illustrative Model of Multi-Component Decay Chains

partial widths for \( \phi_9 \rightarrow \phi_i \phi_j \ (i \geq j) \)

\[
\begin{align*}
\Gamma_{ij}^9(r, s)/\Gamma_{00}^9(0, 0) = & \\
\Gamma_{0j}^9(r, s) &= \frac{\Gamma_{ij}^9(r, s)}{\Gamma_{0j}^9(0, 0)}
\end{align*}
\]
Illustrative Model of Multi-Component Decay Chains

tend to minimize kinetic energy and favor asymmetry

partial widths for $\phi_9 \rightarrow \phi_i \phi_j \ (i \geq j)$

$tend to produce light states and favor asymmetry$

$tend to minimize kinetic energy and favor symmetry$

tend to produce lightest states
Illustrative Model of Multi-Component Decay Chains

$s$ increasing

$r$ increasing

$r = -3, s = -4$

$r = -3, s = 0$

$r = -3, s = +4$

$r = 0, s = -4$

$r = 0, s = 0$

$r = 0, s = +4$

$r = +3, s = -4$

$r = +3, s = 0$

$r = +3, s = +4$

$\Gamma_i/\Gamma_0$

$n_h$

$10^{-12}$ $10^{-9}$ $10^{-6}$ $10^{-3}$ $1$ $10^3$ $10^6$

Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors
Illustrative Model of Multi-Component Decay Chains

$s$ increasing

Decays to lightest state occur over similar times
Illustrative Model of Multi-Component Decay Chains

$s$ increasing

$r$ increasing

$r = -3$
$s = -4$

$r = -3$
$s = 0$

$r = -3$
$s = +4$

$r = 0$
$s = -4$

$r = 0$
$s = 0$

$r = 0$
$s = +4$

$r = +3$
$s = -4$

$r = +3$
$s = 0$

$r = +3$
$s = +4$

$t/t_I$

$10^{-3}$
$10^{-2}$
$10^{-1}$

$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$

$0.1$

$1$

$a^3\rho/\left[a^3(t_I)\rho_{tot}(t_I)\right]$

$10^{-3}$
$10^{-2}$
$10^{-1}$

$1$

$10^3$
$10^6$
$10^9$
$10^{12}$

$10^3$
$10^6$
$10^9$
$10^{12}$

$10^3$
$10^6$
$10^9$
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$10^{12}$

Jeff Kost

Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors
Illustrative Model of Multi-Component Decay Chains

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$s$ increasing

$r$ increasing

$w$ increasing

$t/t_I$ increasing

$r = -3$
$s = -4$

$r = -3$
$s = 0$

$r = -3$
$s = +4$

$r = 0$
$s = -4$

$r = 0$
$s = 0$

$r = 0$
$s = +4$

$r = +3$
$s = -4$

$r = +3$
$s = 0$

$r = +3$
$s = +4$
Illustrative Model of Multi-Component Decay Chains

$s$ increasing

$r$ increasing

$s$ increasing

$p/m$ increasing

$g(p, t_{\text{now}})/N = r - 3$
$s = -4$

$g(p, t_{\text{now}})/N = r - 3$
$s = 0$

$g(p, t_{\text{now}})/N = r - 3$
$s = +4$

$g(p, t_{\text{now}})/N = +3$
$s = -4$

$g(p, t_{\text{now}})/N = +3$
$s = 0$

$g(p, t_{\text{now}})/N = +3$
$s = +4$
Illustrative Model of Multi-Component Decay Chains

A variety of distribution functions emerge!
Illustrative Model of Multi-Component Decay Chains

$s$ increasing

$r$ increasing

$T^2(k) \equiv P(k)/P_{CDM}(k)$

$\tilde{g}(k, t_{now})/N$

$s$ increasing
Recall our conjecture:
\[
\frac{\tilde{g}(k)}{N} \approx \frac{1}{2} \left( \frac{9}{16} + \left| \frac{d \log T^2}{d \log k} \right| \right)^{-1/2} \left| \frac{d^2 \log T^2}{(d \log k)^2} \right|
\]

What features can we “resurrect” from this relation?
Illustrative Model of Multi-Component Decay Chains

\[ T^2(k) = \frac{P(k)}{P_{\text{CDM}}(k)} \]

- \( r = -3 \)
- \( s = -4 \)
- \( r = -3 \)
- \( s = 0 \)
- \( r = -3 \)
- \( s = +4 \)

\[ \tilde{g}(k, t_{\text{now}})/N \]

- \( r = 0 \)
- \( s = -4 \)
- \( r = 0 \)
- \( s = 0 \)
- \( r = 0 \)
- \( s = +4 \)

Parameters:
- \( v \)
- \( k [h/\text{Mpc}] \)
- \( v \)
- \( v \)

- \( r \) increasing
- \( s \) increasing

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CONCLUSIONS

• Early-universe processes such as decays within the dark sector can leave identifiable imprints in $f(p)$ and $P(k)$; certain features may allow us to go backwards and archaeologically reconstruct the dark-matter distribution.
  ◦ We found useful analytical tools, such as hot-fraction function $F(k)$.
  ◦ Conjectured relation that can “resurrect” $f(p)$ features from $P(k)$.

• The dark sectors of string theory generically include unstable KK towers similar to the form we have discussed here, leading to multi-modal $f(p)$ distributions and non-trivial $P(k)$ spectra.

• Such approaches may be only probes for dark sector decoupled from SM.
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FUTURE WORK/DIRECTIONS:

• How to incorporate effects that come from SM couplings? Could affect evolution of phase-space distributions in some additional subtle ways.
• Incorporation of observational bounds/constraints (Lyman-α, etc.)
• How do these $T^2(k)$ fall within effective theories of structure formation?
• Addressing the non-linear regime...
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THANK YOU FOR YOUR ATTENTION!